Report

by the

Intercenter

Low Thrust Trajectory Tool Team

Low Thrust (LT) Tool Capability

Gap Analysis

and

State-of-the-Art Tools Assessment

(this is the Product Delivery version, v1.0)

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Introduction

The impetus for this activity has come about from the need to have a tool or set of tools that is available for use at all NASA centers to generate consistent analyses for support of various NASA Headquarters Programs and Projects. The In-Space Investment Area, the new NSI program, and Code S mission support are three examples of customers with needs for consistent analyses being done at various NASA center locations.

The first steps in accomplishing this very large task, is to assess what tools we currently have and what their capabilities and shortcomings are, what features/capability we want in a new common intercenter tool/code or suite of tools/codes, and clearly understand how we get from today's state-of-the-art capability to this new, more robust, more user-friendly, higher fidelity, and quicker turnaround capability -- i.e. a "Gap Analysis".

Purpose

The purpose of this low thrust (LT) trajectory tool activity is to come up with a tool or suite of tools that allows the NASA community to do LT trajectory analyses that is:

1) Consistent (between all centers),
2) Quick turn-around at times (hours/days/week),
3) Rigorousness/fidelity that can be somewhat "dialed in", determined by time allowed: hours/days -- low fidelity; days/weeks -- mid fidelity; week/months -- high fidelity.

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Blue/Bold: Center leads/POCs; Bold: Contributing authors

Customers

This tool or suite of tools will support all customers we may have. For example:

- Code S (all divisions/themes): flight programs, demo flights, mission & technology assessments;
- MSFC ISP Office: transportation/propulsion technology assessments;
- NSI studies;
- RASC/Code M & Code M centers' studies;
- all of our Centers' in-house studies, etc. ... .
- Code R requests;

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This table shows, in essentially a one-page summary (in the original Excel version), many of the pertinent features and capabilities of most state-of-the-art LT tools. This table is meant to be a one-stop "mini-education" on the applicability of all these LT tools as well as their limitations. This table is also meant to be a very concise summary of the much more extensive tool comments shown in Appendix A. Note that the BOLD font in the table is meant to highlight the "Best in Category" for each feature. The "user-friendly" figure of merit (FoM) was rated for each of the existing codes from 1 to 10, with 10 being

Table 1: State-of-the-Art Tools

Low Thrust Software/Code/Tool Comparison/Summary Table

Bold entry is "Best in Category"

Feature: / Code:	Direct/ Indirect	Method / submethod:	User-Friendly	Appli-cation	Turn-around Capability	Easily Converged	Self-starting	Fide-lity
CHEBYTOP	Indirect	Chebyshev polynomial approx. for traj. segments	4	Narrow	Small	Yes	Yes	Low
CHEBYTOP / ss	Indirect	Chebyshev polynomial approx. for traj. segments	5	Narrow	**VeryLarge**	**Yes**	**Yes**	Low
CHEBYTOP/MdlC	Indirect	Chebyshev polynomial approx. for traj. segments	5	Very Narrow	Large	Yes	Yes	Low
QT2 (QuickTOP II	Indirect	Chebyshev polynomial approx. for traj. segments	5	Narrow	Medium	Sometimes	Yes	Low
CHEBYTOP/func/s	Indirect	Chebyshev polynomial approx. for traj. segments	5	Very Narrow	Large	Yes	Yes	Low
SEPSPOT	Indirect	Averaging techniques for planetocentric problems	**6**	Narrow	Large	Yes	No	Med
VARITOP	Indirect	Variational & optimal control using 2PBVP solution	3	Broad	~Large	Sometimes	No	Med
VARITOP/Jupiter	Indirect	Variational & optimal control using 2PBVP solution	3	~broad	~Large	Sometimes	No	Med
SEPTOP	Indirect	Variational & optimal control using 2PBVP solution	2	Mid	~Large	Sometimes	No	Med
NEWSEP	Indirect	Variational & optimal control using 2PBVP solution	2	Narrow	~Large	No	No	Med
Sail	Indirect	Variational & optimal control using 2PBVP solution	2	Narrow	~Large	Sometimes	No	Med
GALLOP	Direct	Parameter optimization(NPOPT	4	Broad	Large	Sometimes	Maybe	Med
Mystic	Direct	Static / Dynamic Control	**6,w/ GUI**	**Very Broad**	~medium	Sometimes	No	**High**
OTIS	Direct	Collocation (Hermite-Simpson nodes)	4	Broad	Large	No	No	Med
SNAP (Num. Integr.)	N/A	Not Applicable	5	Mid	Large	Not App	Not App	**High**
ESPAS environment	N/A	Gradient based optimization	5	Broad	Large	Sometimes	Yes	Med
RAPTOR (Earth-Mars)	Indirect	Variational & optimal control using 2PBVP solution w/ a GA to initiate guesses	6	Narrow	Small	Yes	Yes	Low
Copernicus	Direct/ Indirect /Hybrid	Constrained Parameter Optimization (SQP)/Optimal Control using Multi-PBVP	3	very broad	~Large	Sometimes	No	High

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the desired user-friendliness for a finished graphical user interface (GUI). Note that all existing tools do not approach the desired level of user-friendliness, as none of them have user-friendly GUIs. Additional FoMs are shown in the continued table. As can be seen, the low fidelity tools have greater turn-around capability at the expense of being applicable to fewer types of missions and analyses. Of course, much of this information / data is subjective to some extent, but the LTTT Team has attempted to come to a consensus on all data shown. There are plans in the future to possibly expand this "mini-tutorial" with a number of additional FoMs (J.Condon/JSC) should a need be identified.

Table 1: State-of-the-Art Tools (continued)

JPL GRC JSC

Bold entry is "Best in Category"

Feature: / Code:	Direct/ Indirect	Method / submethod:	3-body, N-body	Multi-Leg, Int Bdy Flyby	Tour Capability	Out-of-Plane?	Documentation
CHEBYTOP	Indirect	Chebyshev polynomial approx. for traj. segments	No / No	No / No	No	No	~3 pg UG/X
CHEBYTOP / ss	Indirect	Chebyshev polynomial approx. for traj. segments	No / No	No / No	No	No	~3 pg UG/X
CHEBYTOP/MdlC	Indirect	Chebyshev polynomial approx. for traj. segments	No / No	Yes/Yes, 1	No	No	~3 pg UG/X
QT2 (QuickTOP II	Indirect	Chebyshev polynomial approx. for traj. segments	No / No	No / No	No	No	Ug/Xm p/Thry
CHEBYTOP/func/s	Indirect	Chebyshev polynomial approx. for traj. segments	No / No	No / No	No	No	~3 pg UG/X
SEPSPOT	Indirect	Averaging techniques for planetocentric problems	NotAppl.	Not Appl.	No	Yes	~200pg
VARITOP	Indirect	Variational & optimal control using 2PBVP solution	No / No	Yes/Yes, <3	No	Yes	U/X/T
VARITOP/Jupiter	Indirect	Variational & optimal control using 2PBVP solution	No / No	Yes/Yes, <3	~Yes	Yes	0 pg
SEPTOP	Indirect	Variational & optimal control using 2PBVP solution	No / No	Yes/Yes, <3	No	Yes	??
NEWSEP	Indirect	Variational & optimal control using 2PBVP solution	No / No	Yes/Yes, <3	No	Yes	0 pg
Sail	Indirect	Variational & optimal control using 2PBVP solution	No / No	No? / No?	No	Yes	UG
GALLOP	Direct	Parameter optimization(NPOPT	No / No	Yes/Yes, N	Yes	Yes	UG/X
Mystic	Direct	Static / Dynamic Control	**Yes/Yes**	**Yes/Yes**	**Yes**	**Yes**	**U/X/T**
OTIS	Direct	Collocation (Hermite-Simpson nodes)	Yes?/ No	Yes/Yes, n	Yes	Yes	**Ug/Xm /Th**
SNAP (Num. Integr.)	N/A	Not Applicable	Yes / Yes	Yes/Yes, n	Yes	Yes	u/x/t
ESPAS environment	N/A	Gradient based optimization	Yes?/ No	yes/yes, n	Yes	Yes	UG/X
RAPTOR (Earth-Mars)	Indirect	Variational & optimal control using 2PBVP solution w/ a GA to initiate guesses	*No / No*	No / No	No	Yes	Ug/Xm p/Thry
Copernicus	Direct/ Indirect /Hybrid	Constrained Parameter Optimization (SQP)/Optimal Control using Multi-PBVP	Yes / Yes	Yes / Yes, ?	Yes	Yes	90 pg, Theory (2 pprs)

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State-of-the-Art Methods

A number of methods in use by current state-of-the-art (SoA) tools are listed below with some description of the method and, in instances where available, its application in the tool where it is used. Note that information is continuing to be gathered for this section -- especially as the team becomes aware of other SoA algorithms and techniques.

Optimal Power Determination by Analytic and/or Transversality Conditions:

Both the CHEBYTOP and VARITOP tools use this method to determine optimal power for a trajectory.

Calculus of Variations Method*:

A branch of mathematics which is a sort of generalization of calculus. Calculus of variations seeks to find the path, curve, surface, etc., for which a given function has a stationary value (which, in physical problems, is usually a minimum or maximum). Mathematically, this involves finding stationary values of integrals of the form

$$I = \int_b^a f(y, y', x)\,dx \qquad (1)$$

I has an extremum only if the Euler-Lagrange differential equation is satisfied, i.e., if

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0 \qquad (2)$$

the fundamental lemma of calculus of variations states that, if

$$\int_b^a M(x) h(x)\,dx \qquad (3)$$

for all h(x) with continuous second partial derivatives, then

$$M(x) = 0 \qquad (4)$$

on (a, b).

*http://mathworld.wolfram.com/CalculusofVariations.html

Collocation Methods**:

A method of determining coefficients α_l, in an expansion:

$$y(x) = y_0(x) + \sum_{l=1}^{q} \alpha_l y_l(x)$$

so as to nullify the values of an ordinary differential equation L[y(x)] = 0 at prescribed points.

**http://mathworld.wolfram.com/CollocationMethod.html

Legendre Pseudospectral Method:

A Legendre Pseudospectral Method for trajectory optimization, proposed by Mike Ross and Fariba Fahroo of the Naval Postgraduate School, has been applied to many space vehicle trajectory problems, including low thrust transfer problems such as an Earth-Mars transfer. The method can handle many types of constraints, and can handle variable Isp and power problems.

The method uses a Legendre pseudospectral differentiation matrix to discretize nonlinear differential equations (such as the equations of motion) into nonlinear algebraic equations. The equations are then posed in the form of a nonlinear optimization problem and solved numerically as a parameter optimization problem. The method has been demonstrated to work very well with both continuous and discontinuous states and controls. One benefit of the method is that the co-states can be extracted from the solution of the numerical optimizer.

Static/Dynamic Control Method:

Static/Dynamic Control (SDC) is a new, general optimization algorithm that was derived to address a class of problems with the same structure as low-thrust optimization. SDC best fits into the direct method category. However, unlike other direct methods, the explicit time dependence of the optimization problem is not removed by parameterization. The SDC optimization algorithm is a form of optimal control. The SDC optimization algorithm is based in part on the Hamilton, Bellman, Jacobi dynamic programming equation <REF>. Unlike traditional differential dynamic programming methods, SDC is constructed to solve highly nonlinear and non-convex problems with a dual dynamic and parametric structure. Optimal solutions generated by SDC satisfy both the necessary and sufficient conditions of optimality.

<REF>: Kirk, D.E., *Optimal Control Theory: An Introduction*, Prentice-Hall Inc., N.J., 1970.

Pontryagin Maximum Principle:

Pontryagin Maximum Principle combined with Sequential Quadratic Programming and Nonlinear Multi-Dimensional Boundary Value Problem Solvers

Any spacecraft trajectory design and optimization problem can be posed as an infinite dimensional optimal control problem whose solution requires a solution to a multi-dimensional multi-point boundary value problem. A generalized architecture based on this method allows the specification of any cost function, any set of target conditions, in any force field model. Depending on the nature of the problem, several different numerical based algorithms are used to obtain solutions. For a design problem without optimization, targeting algorithms are used where conditions can be placed along various parts of the trajectory. For an optimization problem, there are three solution methods: 1) if the control is discretized to produce sub-optimal solutions, an explicit optimization is made on a general cost function with a nonlinear parameter optimization algorithm, 2) if the optimal control problem is solved by adjoining the state equations to the Hamiltonian function via time varying Lagrange multiplier vectors and the associated transversality conditions are treated as constraints, the problem is solved as a multi-dimensional boundary value problem, 3) if the transversality conditions are not used so that only the kinematic boundary conditions are treated as initial, intermediate, or final constraints, the problem is solved as a parameter optimization problem where the initial and/or intermediate values of the Lagrange multiplier vector are treated as parameters.

In any case efficient nonlinear root finders and nonlinear programming methods, such as the Sequential Quadratic Programming algorithms are used to extremize the given cost function that is dependent on the independent parameters identified for a specific problem. One of the systems identified in this report (COPERNICUS) uses this architecture. The emphasis on the system is directed more towards the architecture, the force models, the propulsion systems, and the target conditions. Regardless of the problem, it is always cast as a multi-dimensional root finding problem or a parameter optimization problem. The equations of motion are not discretized, but instead are treated as differential constraints.

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Desired Goal(s)

Example of what we want:

New tool:

Easily converged w/ little or no starting guesswork
- Report “reasonable” sub-optima as well as global optimum, on request

Rapid turn-around capability w/ trade data generation
- Parameter optimizations & scanning, sufficiently rapid trade-space sweeps

Medium to high fidelity, as desired
- Numerical integration of physical trajectories or equivalent
- High-quality ephemeredes
- Multi-body g fields* & planetary systems, incl. moons, as appropriate or as selected
- Flexible capability to model power & propulsion systems* & their operating margins
- High fidelity option to include SRP, shadowing, & closed loop guidance*
  (if necessary or desired)
- Flyby / Gravity assist on/off capability
- 3-body on/off capability

Broadest application possible
- All relevant low-thrust systems, i.e. NEP, SEP, M2P2, Sails
- Gravity assist on/off capability, control across gravity fields* (if necessary)
- Model entire trajectory, including spirals & Kepler arcs (coasts, final conditions, etc.)
  - Missions:
    - Heliocentric/interplanetary
    - Planetocentric
    - Interstellar
    - Tours/multi-primary
    - Restricted 3 [Libration Point missions (any 2 primaries)], 4, 5, ..., n body problems
    - Multi-spacecraft
    - Lunar Missions including ballistic lunar capture
  - Propulsion options
    - SEP
    - NEP
    - Solar Sails
    - Mag-Sails
    - Gravity assist on/off capability
    - Control across gravity fields* (if necessary)
    - Chemical (monopropellant, bi-propellant, cold gas, etc., as applicable)
    - Multiple propulsion systems/hybrid models
    - Combined powered (low thrust) with gravity assist(s)

On-line editable data bases w/ automated backup
- Standard and/ reference test cases* all with both sample input and output listings
- Library of cases that can be edited/customized & saved

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User-friendly GUI I/F
- With problem setup aids (e.g. Wizards)
- Interactive real-time visualization of the solution process
- Real-time data display of iteration/design/convergence process

Sufficient documentation & on-line help
- Stand alone with on-line documentation and help screens
- Detailed theory document with appropriate references
- Comprehensive tutorial with example cases and benchmarks

Multi-platform:
- Wintel (PC)
- Mac OS-X
- Unix / Linux

Releasable in stages/modules
- May be developed in stages, with interim releases
  (e.g. 1st EP, then sails, then Mag-sails)

Done w/ the perspective of a completed S.O.A. review
- State-of-the-art search
- Identification of existing/available usable code/tool modules & suites
- Start w/ preparation of a specification

Non-proprietary (may require ITAR restrictions/controls)
- Published source code with at least skeleton documentation
- Available to qualified users
  (a fee covering reproduction & distribution costs may be appropriate)
- To qualify, a user should need only to demonstrate compliance w/ applicable ITAR restrictions.

Disciplined process:
- MIL-STD 498, SEI-CMM, NASA STD 2100-91, or similar
- Configuration control w/ distribution of authentic versions
  (w/ a non-proprietary system, custom versions may exist but there must be a controlled distribution of authentic versions)
- Rigorous standards testing w/ non-proprietary software testing tools
  (includes defect tracking, software Q/A, verification, & validation)

* Example of the many topics to be discussed at the NASA Low Thrust TIM, July 16-18, at the LPI, Houston, TX

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Summary of Desired Product(s)

Multi-dimensional (2 - 3) quick turn-around trade-space tool:

Simple(r) tool for trajectory optimization w/o all (but some) of the high fidelity "bells and whistles"
- This tool should be able to run 2- to 3-dimensional trade space assessments
  (at least as well as the **CHEBYTOP**/spreadsheet version -- see Table 1)
- Optimization of ΔV/propellant, power level, and Isp (all 3 concurrently for max. "payload")
- Optimization of trip time, power level, and I_{sp} (all 3 concurrently for max. "payload")
- This tool should also be incorporated into the spreadsheet environments & desktops
  (Team X, Team X/ICEMaker, Team NSI/ICEMaker, our collaborative design centers, ...)
  (Mac [OS X], PC [Windows NT?], and Unix/Linux [Sun?, SGI?, HP?, PC? other?])
- Tool should allow extremization of any trajectory quantity of interest such as the common ones that include maximum payload, minimum transfer time, but other less common ones such as minimum flyby radius, minimum V_∞ at a flyby body, multi-objective cost functions, etc.
- Tool should allow consistent constraints on any trajectory quantity of interest
  (e.g., minimum perihelion, maximum planetary entry speed, etc.) when feasible

High fidelity mission/trajectory tool:

High fidelity tool to do overall optimization of missions for given generic systems
- Optimization of ΔV/propellant, and/or trip time, and/or power level, and/or Isp
  (incl. both spiral & interplanetary portions)
- This would utilize "generic" propulsion system/subsystem/component models
- This tool should be able to run "2-D" trade spaces at least as well as **VARITOP** & **Mystic**
- System should allow constraining of control rates (e.g., thrust vector turn rates should be limited to a maximum allowable value for vehicles with limited turn rate capability), for problems for which it is necessary.

High fidelity system/trajectory tool:

High fidelity tool to do optimization of vehicles (thruster-wise, etc. ...) for a given mission(s)
- Optimization of propulsion (& power?) systems/subsystems/components
  (incl. both spiral & interplanetary portions)
- This would utilize high fidelity propulsion system/subsystem/component models
- This may utilize high fidelity power system/subsystem/component models

High fidelity human missions system/trajectory tool:

High fidelity tool to do optimization of vehicles
- Multiple spacecraft, multiple propulsion systems, and variable Isp features
- Human based constraints:
  lighting, crew schedules, turn rates, g-limits, specific human-driven orbits
- Adaptation/development from a current engineering prototype tool
- System should allow constraining of control rates (e.g., thrust vector turn rates should be limited to a maximum allowable value for vehicles with limited turn rate capability), for problems for which it is necessary.

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Desired Low Thrust Code/Tool Comparison/Summary Table

Bold entry is "Best in Category"

Code: / Feature:	Direct/ Indirect	Method/submethod:	User-Friendly	Appli-cation	Turn-around Capability	Easily Converged	Self-starting	Fide-lity
Desired Tools:								
Multi-dimension quick turn-around tool			**10**	Broad	**Lrg-VLrg**	**Yes**	**Yes**	Lw/Md
High fidelity mission / trajectory tool			9	**Broad**	Med-Lrg	Yes	Yes	**High**
High fidelity system / trajectory tool			9	**Broad**	Med-Lrg	Yes	Yes	**High**

Desired Low Thrust Code/Tool Comparison/Summary Table

Bold entry is "Best in Category"

Code: / Feature:	Direct/ Indirect	Method/submethod:	3-body, N-body	Multi-Leg, Int Bdy Flyby	Tour Capability	Out-of-Plane?	Documentation
Desired Tools:							
Multi-dimension quick turn-around tool			Yes/No	Yes/Yes,9	**Yes**	**Yes**	UG/X/T
High fidelity mission / trajectory tool			**Yes/Yes**	**Yes/Yes,9**	**Yes**	**Yes**	UG/X/T
High fidelity system / trajectory tool			Yes/Yes	Yes/Yes,9	**Yes**	**Yes**	UG/X/T

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Conclusion

This document is a thorough review of the current state of the art in low thrust analysis tools. The need for more capable overall mission/trajectory modeling tools and systems modeling and analysis is evident as a result of the needs by our customers and the state in In-Space technologies for propulsion. This document will lay the foundation for the steps that need to be taken over the next two years (FY03 & FY04) to bring this analysis capability up to the level that is necessary to support decisions for implementation of technology development for advanced In-Space Propulsion.

The proposed suite of 3-4 tools will cover most if not all needs for LT mission analysis for the near future. The idea of multiple high fidelity tools, is that one tool cannot do everything, and a complementary set of capabilities in those high fidelity tools make sense. It is still envisioned, though, that the high fidelity tools would be able to "jump start" each other through a file (formatted appropriately) with common data "hooks". An example of this would be that the high fidelity mission/trajectory tool would output a "jump start" file that the high fidelity mission/systems tool would use to converge to its first solution(s).

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MYSTIC User's Guide for Version 6.6, Whiffen, G. J.

Preliminary Design of Low-Thrust Interplanetary Missions (AAS 99-338)

An Approach to Design and Optimization of Low-Thrust Trajectories with Gravity Assists (AAS 01-468)

Debban, T., McConaghy T., and Longuski, J., "Design and Optimization of Low-thrust Gravity Assist Trajectories to Selected Planets", AIAA/AAS Astrodynamics Specialist Conference, Paper No. AIAA 2002-4729, Monterrey, CA, August 5-8, 2002.

Whiffen, G. J. and Sims, J. A., "Application of the SDC Optimal Control Algorithm to Low-Thrust Escape and Capture Trajectory Optimization," AAS/AIAA Space Flight Mechanics Meeting, Paper No. AAS 02-208, San Antonio, Texas, January 28-30, 2002.

Sims, J. A., Whiffen, G. J., Finlayson, P. A., and Petropoulos, A. E., "Low-Thrust Orbit Transfer around Minor Planets," AAS/AIAA Space Flight Mechanics Meeting, Paper No. AAS 02-139, San Antonio, Texas, January 28-30, 2002.

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Ranieri, C., Ocampo C., High Power Variable Specific Impulse Time Constrained Roundtrip Trajectories to Mars and Jupiter. in Preparation for the AAS/AIAA Astrodynamics Specialist Conference, August 2003 (exact title may change slightly).

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Lunar Sample Return via the InterPlanetary Superhighway
M. Lo and M. Chung, California Institute of Technology, Pasadena, CA

New methods/algorithms (2002 ASC):

AIAA 2002-4730
Low-Thrust Interplanetary Mission Design Using Differential Inclusion
J. Hargens and V. Coverstone, Univ. of Illinois at Urbana-Champaign, Urbana, IL

AIAA 2002-4894
Constrained Circularization in Elliptic Orbit Using Low-Thrust With Shadowing Effect
J. Kechichian, The Aerospace Corp., Los Angeles, CA

AIAA 2002-4895
A Method of Efficient Inclination Changes for Low-Thrust Spacecraft
R. Falck and L. Gefert, NASA Glenn, Cleveland, OH

AIAA 2002-4896
Optimal Low-Thrust Out-of-Ecliptic Trajectories
J. Bader, P. Gurfil and N. Kasdin, Princeton Univ., Princeton, NJ

AIAA 2002-4900
Simple Control Laws for Continuous-Thrust Escape or Capture and Their Use in Optimisation
A. Petropoulos, G. Whiffen and J. Sims, JPL, Pasadena, CA

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Compensator Design for Low-Thrust Interplanetary Missions
D. Fiehler and C. Kluever, Univ. of Missouri, Columbia, MO

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B. Dachwald and W. Seboldt, DLR, Cologne, Germany

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A. Trask, W. Mason and V. Coverstone, Univ. of Illinois at Urbana-Champaign, Urbana, IL

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K. Chen, T. McConaghy, M. Okutsu and J. Longuski, Purdue Univ.

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G. Rauwolf and A. Friedlander, Science Applications I'tnl Corp.; K. Nock, Global Aerospace Corp., CA

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N. Hamilton, US Air Force Academy; D. Folta and R. Carpenter, NASA GSFC

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J. Prussing, Univ. of Illinois at Urbana-Champaign, Urbana, IL

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L. Casalino and G. Colasurdo, Politecnico di Torino, Torino, Italy

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M. Macdonald and C. McInnes, Univ. of Glasgow, Glasgow, United Kingdom

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Solar Sail Orbit Operations at Asteroids: Exploring the Coupled Effect of an Imperfectly Reflecting Sail and a Non-spherical Asteroid
E. Morrow, California Space Institute; D. Scheeres, Univ. of Mich.; D. Lubin, California Space Inst.

AIAA-2002-4992
Solar Sail Dynamics and Control Using a Boom Mounted Bus Articulated by a Bi-State Two-Axis Gimbal and Reaction Wheels
E. Mettler and S. Ploen, JPL, Pasadena, CA

Some historical papers of (possible) interest/applicability:

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AIAA-2002-4525
A Wind Trajectory Design Incorporating Multiple Transfers Between Libration Points
H. Franz, Computer Sciences Corp., Greenbelt, MD

ToC

Appendix A: State-of-the-Art Tool Assessment

Detailed Strengths/Weakness (Pro/Con) Descriptions of Existing State-of-the-Art Tools

Examples of what various tools we currently do have:

CHEBYTOP: Chebyshev (Polynomial) Trajectory Optimization Program

Some of the strong points of this tool/code:

+ ~User-friendly simple tool

  CHEBYTOP has only the usual old style namelist input, but with a limited set of inputs, and with limited options for many of those inputs, this tool is as user-friendly as the current set of codes get.

+ Large turn-around capability

  Run times with CHEBYTOP are very fast due to its less rigorous modeling and analysis techniques, which contribute greatly to having the capability for "wrapping" the tool for large multi-dimensional trade-space type analyses which require large turn-around capability. This has been accomplished in a number of ways:

  1) The tool was put into a spreadsheet environment and wrapped with Visual Basic for Applications to give a capability to do 100's to 1000's of single trajectory runs to determine various types of optima (Isp, trip time, etc) for a range of missions (i.e. a "trade space").
  2) The tool was put into a spreadsheet environment as a function call to give a capability to do dozens of single trajectory runs to determine various types of optima (Isp, trip time, etc) for a range of missions.
  3) The tool was wrapped into a "QuickTOP 2 driver" to give a capability to do many, many single trajectory runs and trade spaces for a range of missions.
  4) The tool was wrapped into a ModelCenter environment to give a capability to patch two separate trajectories into a single trajectory with an unpowered swingby of an interim body.

+ Easily converged

  CHEBYTOP easily converges in most cases that it is applicable for, however a user MUST still know enough about expected results to be aware of erroneous results, since there are a number of limitations one has when using this tool (e.g. out-of-plane and T/W limits).

+ Self starting

  CHEBYTOP returns converged results with only a minimal number of user inputs.

Some of the weak points of this tool/code:

- Needs some starting guesses

  CHEBYTOP has to have sufficiently close starting guesses on a few of the input parameters, or the output will be useless -- sometimes incorrect results are not the easiest things to recognize.

- Low fidelity

  CHEBYTOP uses a coarse Chebyshev polynomial approximation for various segments of the trajectory, and does not ever do an integration of the results/trajectory in any way.

- Little documentation

  The extent of the CHEBYTOP documentation is a few page user's guide with some old 1960's reports on the early versions. QuickTOP 2 was documented to a great extent compared to other CHEBYTOP versions.

- No flybys (w/o ModelCenter or other "wrapper")

  There is no capability to do interim body flybys, except for option #4 above.

- No out-of-plane missions (Pluto Orbiter)

  CHEBYTOP was not formulated properly to do missions more than a few degrees out of the ecliptic. This does include all primary bodies except Pluto, and flyby missions can still be done with Pluto as a destination.

- Narrow application

  The narrow extent of CHEBYTOP's applicability limits it usefulness (requires near ecliptic trajectories and T/W ranging from 10-4 to 10-6 G's).

- Interplanetary trajectory only

  CHEBYTOP does only the interplanetary portion of the trajectory, using the Edelbaum approximation for spiral up or down at a solar system body.

- No "tour" capability

  With no capability to do interim flybys, there is also no capability to do multi-body tours.

QuickTOP: New (1990) QT2 Driver for the CHEBYTOP System

Some of the strong points of this tool/code:

+ …

Some of the weak points of this tool/code:

- …

ToC

VARITOP: Variational Calculus Trajectory Optimization Program

SEPTOP: (VARITOP-based) Solar Electric Propulsion Trajectory Optimization Program

NEWSEP: New (VARITOP) Solar Electric Propulsion Trajectory Optimization Program

Sail: VARITOP customized for Solar Sails

SEPTOP and NEWSEP are essentially the same as VARITOP except for the model of the thruster. SEPTOP models the thruster with polynomials (4^{th} order) for thrust as a function of power and mass flow rate as a function of power. NEWSEP models the thruster with discrete throttle steps for thrust and mass flow rate. The strong and weak points for SEPTOP and NEWSEP are very similar to VARITOP with the notable exception that NEWSEP can be much more difficult to converge due to the discontinuous nature of the throttle steps.

Some of the strong points of these tools/codes:

+ Higher (i.e. medium) fidelity

  VARITOP takes a huge step in fidelity beyond that of CHEBYTOP due to the more rigorous nature of the algorithm/analysis scheme. Out-of-plane missions, higher T/W missions, atypical control law input capability, and interim body flybys are all possible with this tool.

+ Medium turn-around capability

  VARITOP does still have much capability for one-dimensional trade space analyses with additional options in the namelist input for putting in ranges for a particular independent variable of choice.

+ Little more documentation

  The extent of the VARITOP documentation includes at least two much larger, more well "filled out" user's guides.

+ Does flybys

  As mentioned above, there is the capability to do interim body flybys. There is an option to include up to 3 (or 4? or x?) interim bodies in a trajectory.

+ Much broader application

  As mentioned above, there is the capability to do mission with a number of additional "degrees of freedom" (e.g. the four mentioned above: out-of-plane missions, higher T/W missions, atypical control law input capability, and interim body flybys). There has also been a "Jupiter vicinity" version of VARITOP created where the primary body is Jupiter (replacing the Sun in the original version), with many, if not all of the known Jovian moons modeled in as the system (replacing the solar system in the original code).

+ Has application to other transportation systems (e.g. sails)

  There exists flexibility to put in various thrust direction and magnitude control laws, which allows modeling for other transportation options such as solar sails.

Some of the weak points of this these tools/codes:

- Not user friendly, needs good starting co-states, difficult to converge

  With the additional capability in VARITOP, there is the corresponding greater difficulty for getting results to converge. The need for initial guesses (on the 7) co-states values makes it difficult to generate new analyses. On occasion much time and effort is spent "walking" analyses from a current converged case to another desired case which is yet to be converged on. Fortunately there is the capability to "walk" a solution from one case to another -- if the cases are related closely enough.

- Interplanetary trajectory only

  VARITOP does only the interplanetary portion of the trajectory, using the Edelbaum approximation for spiral up or down at a solar system body. Recent developments, though, include some new features for modeling spirals at the beginning or end of a trajectory. These recent developments still do not effect the mission optimals on Isp or power, though, at this time.

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Mystic:

Some of the strong points of this tool/code:

+ High fidelity

  Fully integrated with multi-body gravity fields (that can be turned off) and high-quality ephemeris. Many other high fidelity modeling features planned to be added.

+ Flybys: Can find its own flybys to improve performance

+ Well developed GUI for inputs and outputs

+ Does not require a good starting guess

+ Extensive User's Guide (does not include recent developments)

+ Optimizes interplanetary and spiraling portions simultaneously as a single integrated trajectory

+ Broad application

Some of the weak points of this tool/code:

- No automated parametric study capability has been implemented

- Computationally intensive

ToC

GALLOP: Gravity Assisted Low-thrust Local Optimization Program

GALLOP is currently under development. A very preliminary version exists.

Some of the strong points of this tool/code:

+ Multiple flybys

  The basic structure of the trajectory propagation was designed so that convergence would be more robust (than SEPTOP) for trajectories that include gravity assists.

+ Intuitive variables

  The independent variables are more physically intuitive than some of the co-states required for the Calculus of Variations methods (e.g., SEPTOP). This typically simplifies generating good initial guesses.

+ Preliminary GUI is available for starting program (initial guesses) and analyzing the results.

+ Documentation: preliminary User's Guide

+ Broad application

  Out-of-plane missions. Multiple flybys. Can optimize impulsive trajectories. Could eventually model many types of low-thrust propulsion systems.

Some of the weak points of this tool/code:

- Lower fidelity (than SEPTOP)

  Trajectories are propagated with conic arcs (low thrust is modeled as small impulsive ΔVs). As a strong point, the mission, spacecraft, and propulsion system models are like SEPTOP.

- Interplanetary trajectory only (plan to use same spiraling approximation as SEPTOP)

- No automated parametric study capability has been implemented (plans for 3-D trade space capability)

- Many features yet to be added.

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OTIS/SNAP: Optimal Trajectories by Implicit Simulation/SNAP

Some of the strong points of these tools/codes:

OTIS Description:
OTIS is one of two widely used trajectory optimization programs within NASA and the aerospace community in general. The implicit integration technique within OTIS is one of the best means of obtaining optimal for a wide range of earth to orbit and other space missions. With OTIS, one can model missions that branch into two or more separate trajectories and missions with end point, mid point, and path constraints. OTIS also provides a very general means of modeling vehicle system model. It is well documented and in wide spread use. Its central force model is quite general and includes an interplanetary capability. There are several sets of equations of motion that are particularly well suited to low thrust missions as well.

SNAP Description:
SNAP is a GRC developed high fidelity trajectory/orbit propagator that provides the user a great measure of freedom in mission modeling. User specifiable features include planetary N-body forces, solar radiation pressure, atmospheric drag, and thrust and steering modeling, high order gravity field calculations, shadowing, and orbit averaging, deep well spiraling, and limited optimization for non-continuous thrust cases. An RKF 7/8 order variable step size integrator is used for accuracy.

+ …

Some of the weak points of these tools/codes:

- …

SEPSPOT: Solar Electric Geocentric Transfer w/ Attitude Constraints Program for Optimization of Trajectories

Some of the strong points of this tool/code:

+ …

Some of the weak points of this tool/code:

- …

ESPAS: Environment for Spacecraft Propulsion Analysis and Sizing

Some of the strong points of this environment:

+ …

Some of the weak points of this environment:

- …

ToC

RAPTOR: RAPid Trajectory Optimization Resource

Some of the strong points of this tool/code:

+ User-friendly tool

  Has a GUI user interface for inputs and outputs.

+ Can work a mission in reverse to find the necessary initial conditions.

  Many times when a mission is initially developed, the final conditions (like mass) are known. With RAPTOR, these final conditions can be specified and the required initial conditions calculated.

+ Can optimize or scan vehicle and mission parameters of interest, such as mass, power, Isp, and heliocentric transfer time.

+ Uses a genetic algorithm to generate the initial guesses for the optimization parameters.

  This saves time and allows a larger solution space to be search.

+ Modular in design.

  New models and/or algorithms can be added or exchanged without totally rewriting the program.

+ Well documented code and users guide.

  The code has been documented to aid other users in making changes or additions. A user guide is being written (80% complete) with comments and instructions on using RAPTOR, and the theory behind the various algorithms.

+ Roundtrip missions can be optimized.

Some of the weak points of this tool/code:

- Long run times

  Probably the biggest weakness. The run times can be lengthy.

- No flybys

  There is no capability to do interim body flybys.

- Optimizes trajectory controls for the interplanetary transfer only.

  Planetary spirals assume the thrust vector is aligned with the vehicle's velocity vector. The controls for the interplanetary trajectories are fully optimized.

- Power and thruster models are very basic.

  Power and Isp are held constant for the entire mission.

- Limited to a thrust-coast-thrust interplanetary sequence

Both a + and - RAPTOR is still being updated and improved.

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Copernicus:

Prototype Development and Complex Space Mission Investigations

Description: Targeted effort to complete a fully functioning prototype version of generalized system for trajectory design and optimization, **Copernicus**, with a well designed graphical user interface and high quality report and data file generation and visualization. The project will also support analysis of complex missions involving multi-body gravity fields such as missions associated with the interior libration point of the Earth-Moon system, long duration thrust arcs, and possibly the use of multiple spacecraft.

Some of the strong points of this tool/code:

System can currently solve:

+ planet centered transfers (reduced orbit, orbit, rendezvous, intercept)
+ lunar transfers (LEO-LMO, ballistic lunar capture)
    a. free return
+ interplanetary transfers
    a. one way
    b. round trip
    c. multi-gravity assists
    d. sample return
    e. cyclers
+ libration point (S/E, E/M, etc.) missions
    Lissajous, halo, figure 8 libration point orbit design
    transfers from LEO to E/M libration points and orbits; leo to S/E libration points
    Earth-Moon libration point to Sun-Earth libration point transfers
+ Any of these trajectories can be solved with impulsive, high thrust finite burn, or low thrust finite burns with systems using constant or variable specific impulse
+ System can be used as a learning tool, as a research tool, and a preliminary and high fidelity mission design and optimization tool
+ System can generate solutions with multi-objective cost functions

Some of the weak points of this tool:

- Still under development
- GUI not yet implemented (development has been initiated)
- Requires user to understand optimization and complex trajectory dynamics in cases where trajectories operate in complex force field
- Some problems require providing a good initial estimate; these problems should have a initial estimator wizard in the GUI

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Appendix B: Reference Mission List

For Low Thrust Tool Check-out & Verification

This is the reference mission list as it currently stands. It will grow or shrink depending on usefulness of the cases listed for covering the entire mission trade-space that can be described as "all missions, all destinations, all transportation options, and all reasonable trip times". Once the new suite of tools is checked out with this set of missions, the LTTT Team will know what missions should remain in this reference list. Currently, it is believed this set of 31 missions covers the entire mission trade-space sufficiently.

1) Classic minimum time to Mars, circular coplanar
2) Earth - Mars flyby
3) Earth - Mars rendezvous
4) Earth - Mars flyby - Vesta flyby
5) Earth - Mars flyby - Vesta rendezvous
6) Earth - Jupiter flyby
7) Earth - Venus flyby - Jupiter flyby
8) Earth - Tempel 1 Rendezvous
9) Earth - Venus flyby - Venus flyby - Jupiter flyby - Pluto flyby
10) Earth - [more than 1 rev around the Sun] - Jupiter flyby
11) Earth - Venus flyby - Mercury rendezvous
12) Earth - Tempel 1 Rendezvous - Earth Flyby
13) Mars Sample Return (with 1 spacecraft and with 2 or more S/C)
14) Comet sample return
15) Multiple asteroid rendezvous
16) 1 AU polar (inclined 90° to the ecliptic) orbiter mission
17) 5-years to Jupiter/Europa Orbiter
18) 8-years to Saturn/Titan Orbiter
19) 10-years to Uranus/Titania Orbiter
20) 12-years to Neptune/Triton Orbiter
21) 12-years to Pluto/Charon Orbiter
22) 6-years to Jupiter (Moon) Tour
23) 9-years to Saturn (Moon) Tour
24) 11-years to Uranus (Moon) Tour
25) 13-years to Neptune (Moon) Tour
26) 12-years to Pluto Tour
27) Kuiper Belt-Pluto Explorer
28) Earth moon system (3-body/libration) low thrust
    - LEO to Low Moon Orbit
    - LEO to Low Moon Orbit Roundtrip
    - LEO to Earth-Moon Libration Point(s) and Libration Point Halo Orbit(s)
29) Sun/Earth 3-body libration point mission(s)
    - LEO to Sun/Earth Libration Point Orbit
    - Sun/Earth Libration Point Orbit to Other Sun/Earth Libration Point Orbit
30) MW to GW interplanetary mission(s)
31) Earth/Sun/Moon 4-body mission and/or other "n-body" mission(s)
    - Earth/Moon Libration Point (or orbit) to Sun/Earth Libration Point (or orbit)

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As a follow-on summary of the reference mission list above, the LTTT Team will complete the following matrix to provide a quick reference summary of what tools can to what missions. Many times analysis of particular missions are inappropriate with certain tools. An example of this is CHEBYTOP cannot be used to assess 3-body or n-body missions. Once the team has completed filling in this "reference" it should be a useful look-up tool for the many customers of all the various LT mission analysts at all the various government centers. Currently it is ~60% complete.

Table 2: The Reference Mission Set vs. SoA Tools

Reference Missions for Tool Check-out/Verification

Too I: Reference Missions:																
1) Classic minimum time to Mars, circ/coplanar																
2) Earth - Mars flyby																
3) Earth - Mars rendezvous																
4) Earth - Mars flyby - Vesta (7°) flyby																
5) Earth - Mars flyby - Vesta rendezvous																
6) Earth - Jupiter flyby																
7) Earth - Venus flyby - Jupiter flyby																
8) Earth - Tempel 1 Rendezvous																
9) Earth - Venus/Vns/Jupiter flybys - Pluto flyby																
10) Earth - [>1 rev around the Sun] - Jupiter flyby																
11) Earth - Venus flyby - Mercury (7°) rendezvous																
12) Earth - Tempel 1 Rendezvous - Earth Flyby																
13) Mars Sample Return																
14) Comet sample return																
15) Multiple asteroid rendezvous																
16) 1 AU polar (incl. 90° to the ecliptic) orbiter																
17) 5-years to Jupiter/Europa Orbiter																
18) 8-years to Saturn/Titan Orbiter																
19) 10-years to Uranus/Titania Orbiter																
20) 12-years to Neptune/Triton Orbiter																
21) 12-years to Pluto/Charon Orbiter																
22) 6-years to Jupiter (Moon) Tour																
23) 9-years to Saturn (Moon) Tour																
24) 11-years to Uranus (Moon) Tour																
25) 13-years to Neptune (Moon) Tour																
26) 12-years to Pluto Tour																
27) Kuiper Belt-Pluto Explorer																
28) Earth moon low thrust																
29) Earth Solar libration point mission(s)																
30) MW to GW interplanetary mission(s)																
31) Earth/Sun/Moon 4-body/other "n-body"																
# of reference missions tool is applicable for:	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

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Appendix C: Proposed Low Thrust Web Page Outline

Page will include an area/section also accessible outside the ".gov" extension

1. **Technology of Low Thrust**

    a. Propulsion devices / Transportation options
        i. Ion
        ii. Hall effect
        iii. MPD (Magneto-Plasma-Dynamic thruster)
        iv. PPT (Pulsed Plasma Thruster)
        v. Plasma
        vi. Solar thermal
        vii. Solar Sail
        viii. Mag-sail

    b. Power generation
        i. Solar array technology
        ii. Nuclear
            1. Low power modules (aka RTG)
            2. Space reactors
        iii. Power conversion
            1. Brayton
            2. Rankine
            3. Stirling
        iv. Power distribution

2. **History**

    a. Missions (links?) that have flown
        i. SERT 1 & 2
        ii. ATS-x
        iii. DS-1
        iv. MightySat
        v. EOS-?
        vi. NOVA? (early use of PPT and novel control system)

    b. Survey papers (made available as PDFs)
        i. Kerslake's TN on SERT
        ii. Aerospace Corp. document on earth orbital low thrust

3. **Trajectory and Mission Software**

    a. Earth orbital
        i. SEPSPOT (Solar Electric Geocentric Transfer w/ Attitude Constraints Program for Optimization of Trajectories)
        ii. Aerospace Corp. codes (if we can get them to participate)
        iii. SNAP
        iv. OTIS (Optimal Trajectories by Implicit Simulation)

    b. Interplanetary Programs
        i. CHEBYTOP (Chebyshev Polynomial Trajectory Optimization Program)

ii. VARITOP (Variational Calculus Trajectory Optimization Program)
iii. SEPTOP/NEWSEP (Solar Electric Propulsion Trajectory Optimization Program)
iv. John Sims' code (CL SEP v1.3)
v. Mystic
vi. Purdue Univ. codes (e.g. GALLOP)
vii. DIDO
viii. OTIS
ix. Sail
x. Copernicus
xi. RAPTOR

c. ESPAS Environment (Environment for Spacecraft Propulsion Analysis and Sizing)

4. Trajectory Optimization Techniques and References

a. Direct Methods
   i. Collocation
      1. Direct
      2. Pseudo-spectral
   ii. Parameter

b. Indirect Methods
   i. Variational and Optimal Control using 2PBVP solution
   ii. Averaging techniques for earth orbital problems

5. Repository of Reference Missions and other Interesting Solutions

Content consists of software used, input files, output files, graphic of trajectory, controls, orbital parameters, launch date/arrival date (transfer time), and narrative about the mission +/or problem. E.g. there is a classic minimum fuel circular/coplanar Earth-Mars mission used as a benchmark.

6. Low Thrust Dictionary and Thesaurus

7. Links to other web sites of interest

8. Forum to post questions and answers and lessons learned

9. Info on proven NLP (nonlinear programming) codes and other solvers of interest

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Appendix D: Standard Inputs and Outputs List

Standard inputs and outputs for all cases for other codes to check other results

As a starting point for defining a standard set of input and output variables, we chose to start with the set of those found in the simplest tool in our current suite of low thrust tools, namely CHEBYTOP. This is the portion of the "user's guide" that describes each of the input variables and many of the output variables. This appendix reflects the initial thinking of the group present at the 2001 Low Thrust Technical Interchange Meeting held at the Lunar Planetary Institute near JSC in Houston. Again, this will also grow or shrink depending on usefulness of the variables listed here for documenting common inputs and outputs expected in the 3-4 tools that will make up the new suite of LT tools. Once the new tools are checked out, the LTTT Team will know what variables should remain in this input/output listing.

CHEBYTOP Vehicle Specific Inputs

Parameter	Type	Description
nctopt	i	compute variable thrust solution only (abbreviated output)
nmuop	i	optimal power flag (non zero)
*m0	dp(3)	initial vehicle mass in kg.
*p0	dp(3)	initial vehicle power in kW.
alfa	dp(2)	power and propulsion system mass definer (1) = specific mass alpha (kg/kW), (2) = fixed mass (kg)
*is	dp(3)	low thrust stage Isp in seconds
bb	dp	thruster efficiency coefficient (n.d.) (≈ 0.70)
dd	dp	thruster efficiency coefficient (Isp, sec.) (≈ 0)
npow	i	if 0, simulate nuclear (constant) power default = 0 else, simulate solar (distance varying) power
sap	dp	power devoted to house keeping (kW) dflt = 0.0
array	ch*6	solar panel array name, default = 'ast'
acf	dp	solar array concentration factor, default = 1.0
kt	dp	low thrust vehicle tankage fraction (n.d.) (≈ 0.10)
ks	dp	low thrust vehicle structural mass fraction (n.d.) (≈ 0)
madp	dp	departure system mass jettisoned after launch,(kg), default = 0.0
pmin	dp	minimum allowable input power, kW
pmax	dp	maximum allowable input power, kW, default = 10**10
retro	l	if true, simulate high thrust retro stage at arrival
cisp	dp	retro stage Isp (sec)
kr	dp	retro stage tankage fraction (n.d.)
jetis	l	if true, simulate jettison of high thrust retro stage
dunits	dp	units conversion flag for dd (default = 1.0)
rtilt	dp	solar distance to tilt solar arrays, A.U.
nlv	i	launch vehicle number (see table)
lvh	ch*30	launch vehicle name (see table)
mlv	dp(4)	user supplied launch vehicle parameters (1) = total tanked initial mass (full tanks) (kg) (2) = inert jettison mass (kg) (3) = effective Isp (sec) (4) = dry mass for non-full tanks (kg)

CHEBYTOP Mission Specific Inputs

Parameter	Type	Description
rn	dp	heliocentric revolution count (n.d.) dflt=0 => compute optimal no. of revs.
jdl	i(3)	calendar departure date (year, month, day)
*adate	dp(3)	Julian arrival date bias from epoch of jdl
*jdate	dp(3)	Julian departure date bias from epoch of jdl
*tend	dp(3)	mission duration relative to jdate or jdl (days)
shota	ch*14	departure planet name, asteroid number, or 'orb1'
bulsi	ch*14	arrival planet name, asteroid number, or 'orb2'
orb1	dp(~~8~~6)	user supplied orbital elements of departure orbit
orb2	dp(~~8~~6)	user supplied orbital elements of arrival orbit
		(1) = semi-major axis (A.U.)
		(2) = eccentricity
		(3) = inclination (deg.)
		(4) = longitude of ascending node (deg.)
		(5) = argument of perigee (deg.)
		(6) = mean anomaly (deg.)
gmp	dp(2)	gravitational constant of departure and arrival planets (use w/ orb1 or orb2)
rpl	dp(2)	radius of dept and arrival planets (use with orb1 or orb2)
flyby	l	if true, simulate planetary flyby else, simulate planetary rendezvous
nv1	i	departure velocity bias flag
		0, no bias, departure body's velocity is used
		1, simulate asymptotic velocity bias
		2, simulate hyperbolic velocity bias
		3, simulate tangential spiral from depart. planet
nv2	i	arrival velocity bias flag
		0, no bias, rendezvous type arrival vhp = 0.0
		1, simulate asymptotic velocity bias
		2, simulate hyperbolic velocity bias
		3, simulate tangential spiral at arrival planet
nb1	i	departure date flag
		= 0, compute optimal travel angle solution
		= 1, compute optimal date to minimize J (if nb1 = 1,the program assumes nb2 is 1)
		= 2, use departure date as supplied by jdl
nb2	i	arrival date flag
		= 0, compute optimal travel angle solution
		= 1, compute optimal date to minimize J (if nb1 = 1,the program assumes nb2 is 1)
		= 2, use arrival date as supplied by jdl & tend
re	dp(2)	parking orbit radius of perigee & apogee @ departure planet
alt	dp	departure circular parking orbit altitude (km)
ra	dp	arrival parking orbit radius of apogee (km)
rp	dp	arrival parking orbit radius of perigee (km) (or arrival altitude specified as multiple of arrival planet's radius, depending on the relative size of the number, ie 2.5 vs 200, but the exact threshold is unknown)
~~alt2~~ alta	dp	arrival circular parking orbit altitude (km) Note: rp and alta cannot both be non-zero; pick one and set the other = 0
*vhl	dp(3)	departure excess velocity
*vhp	dp(3)	arrival excess velocity

CHEBYTOP Run Descriptive Inputs

Parameter	Type	Description
head	ch*96	run title
acname	ch*12	names of asteroid/comet files (PC version)
acpath	ch*24	path for asteroid/comet ephemeris files (PC version)
t0	dp	trajectory output time bias(days), default = 0.0
delpo	dp	trajectory printout interval in days , default = 20.0
copla	l	coplanar solution flag default = f
choice	i	reference time units flag , default = 1 0 = years, tend and tfl in years 1 = years, tend and tfl in days 2 = canonical, tend, and tfl in days

CHEBYTOP Output Description

The following are the constrained thrust (constant Isp, etc.) trajectory performance results. The outputs / variables are:

tl	Difference between the reference epoch (jdl) and departure
ta	Difference between the reference epoch and the arrival date
tend	The total trip time in days
pe	The input power to the system (kW) (either user specified or optimal)
sap	System auxiliary power (kW)
a0	Initial vehicle acceleration (mm/day^2)
is	Specific impulse of the low thrust system (seconds)
eff	Thruster and power processing system efficiency (kw input/kw output)
alpha	Specific mass of the power and propulsion system
jc	Constrained thrust J (m^2/sec^3 i.e. W/kg)
tp	Total thruster "on" time including spiral times (days)
m0	Initial vehicle mass (kg)
mf	Final vehicle mass at destination (kg)
mp	Total low thrust propellant mass used for the trajectory (kg)
mps	Total mass of the power system and propulsion system (kg)
mn	Total net usable space craft mass (kg)
mt	Total tankage mass of the low thrust vehicle (kg)
ms	Total structural mass of the low thrust vehicle (kg)
tf	Total mission duration (days) (tescape + tend + tcapture)
p	Total effective power output of the low thrust propulsion system

m0/pj, mf/pj, mp/pj, m0/pe, mf/pe, mp/pe are the ratios indicated. These can be helpful in characterizing the system performance and predicting the performance of similar systems. m/m0 is mf/m0.

off on off on --- are the approximated heliocentric thruster switching times (days)

The escape and capture arrival data are: the approximate time duration of the spiral maneuver; m0, the initial mass(kg); m, the final mass(kg); mp, the propellant consumed(kg).

The heliocentric longitude and latitude of the outgoing and arriving excess velocity vectors are “al1, be1, al2, and be2” and are measured in degrees.

ToC

Appendix E: Documented SP-210

(To be updated by LTTT Team over FY03)

Title: **"Electric Propulsion Mission Analysis: Terminology and Nomenclature", 1969, J. P. Mullin, et al, ... J. M. Horsewood, ... C. G. Sauer Jr.** (10-page booklet)

Purpose: Review and add any necessary updates to NASA SP-210 to get a start on our standard variables / datasets, standard mission definitions, standard data dictionary, and reflect any changes in current standard low thrust conventions used in state-of-the-art analyses and tools.

Updated by the

Intercenter (MSFC, GRC, JPL, JSC)

Low Thrust Tool Development Team

for the

NASA In-Space Propulsion Project Office
and Code S, SSE Office

March 20, 2003

NASA SP–210

Electric Propulsion Mission Analysis

Terminology
&
Nomenclature

Prepared by
NASA Office of Advanced Research and Technology
NUCLEAR ELECTRIC PROPULSION SYSTEMS ANALYSIS TASK GROUP

Scientific and Technical Information Division
OFFICE OF TECHNOLOGY UTILIZATION 1969
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Washington, D.C.

FOREWARD

During the past decade, a large number of low-thrust mission studies have been carried out both in the United States and abroad. For a variety of reasons, these studies have displayed an imaginative diversity in terminology-placing an additional burden on the reader. When, in the spring of 1968, NASA established a task group on nuclear electric propulsion systems analysis, which involved the rapid exchange of large quantities of information, the need for a common terminology became more sharply focused. The results of the efforts of that task group to establish such a language are displayed in this document with the suggestion that active workers in the field consider its adoption in their future work.

The International System of Units, as defined in NASA SP-7012, was used throughout. Where considerations of tradition or understanding were felt to predominate, other units were added in parentheses.

The members of the task group who developed, adopted, and agreed to promulgate this information are listed on the following page. Suggestions for future revisions should be directed either to the secretary or to the chairman of that group.

J. P. Mullin
Office of Advanced Research and Technology

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INTRODUCTION

In establishing a set of system parameters for use in mission studies, it is first necessary to define a consistent set of terms and basic relations for reference. Since many groups perform such studies, and since electric propulsion hardware development is intimately associated with software development and mission design, it is prudent to agree on a common terminology. What is attempted here is the formulation of one such language.

SPACECRAFT MASS

Spacecraft initial mass m_o is defined as consisting of the sum of the following masses: low-thrust propulsion system m_{ps}, expendable low-thrust propellant m_p, tankage m_t, retro system m_r, net spacecraft m_n, and structure m_s. Refer to Figure 1 for a typical example of spacecraft mass allocation.

Net spacecraft mass, m_n, is the quantity that has been most often maximized in low-thrust trajectory studies. Net spacecraft mass includes various engineering systems such as guidance, thermal control, attitude control, telecommunications, and supporting structure, as well as mission or science payload mass, m_L. Net spacecraft mass is also occasionally, and somewhat ambiguously, identified as "payload mass." The ambiguity arises in missions where m_n is equivalent, for comparison purposes, to a ballistic spacecraft which is in turn viewed as "launch vehicle payload." We shall, here, demand only that the relationship $m_n \geq m_L$ holds, and that m_L, when used, be carefully defined.

The structural mass, m_s, definition may contribute to the ambiguity of the net spacecraft mass definition in the previous paragraph. In some cases, it is assumed proportional to m_o or m_n and handled separately in the analyses; in other cases, it is not explicitly considered. Because structural mass is inherently included in most subsystem mass allocations, for example, $m_{ps}(\alpha)$, $m_t(k_p)$, $m_r(k_r)$, and $m_n(k_n)$, the latter approach avoids a double penalty. Therefore, the task group recommends that in future work m_s be set equal to zero.

That portion of the initial mass defined as propulsion system mass, m_{ps}, includes both the power, m_w, and the thrust, m_{ts}, subsystems, not including propellant tankage but including all internal structure, mechanisms, cabling, thermal control, and so forth. Refer to Figure 2 for a system schematic of the propulsion system. Defined in this way, m_{ps}, m_w, and m_{ts} are ordinarily considered directly proportional to power, the proportionality constant a being a figure of merit in hardware development. The power subsystem mass, m_w, includes primary power, conversion system, structure, mechanisms, shielding, cabling, mission-peculiar thermal control, and the like. This mass is usually treated as a direct function of power level but may include mission-peculiar anomalies. The mass of the thrust subsystem, m_{ts}, includes thrusters, power-conditioning, vaporizers, isolators, actuators, structures, and so forth. This mass is also a function of power level.

The propellant tankage mass, m_t, follows the classical definition of propulsion inerts, being directly proportional to propellant mass. It includes tankage, residuals, reserves, and propellant expulsion elements. In mission performance analysis, expendable propellant mass is evaluated for each specific mission and does not include propellant reserves or residuals. Propellant boil-off, if any, must be included at some point in the analysis as part of the expendable propellant mass.

In many of the missions to be considered, a chemical retro-propulsion system may be required. If included, retro-propulsion system mass, m_r, is made up of two components: first is a

FIGURE 1. Typical Mass Allocation

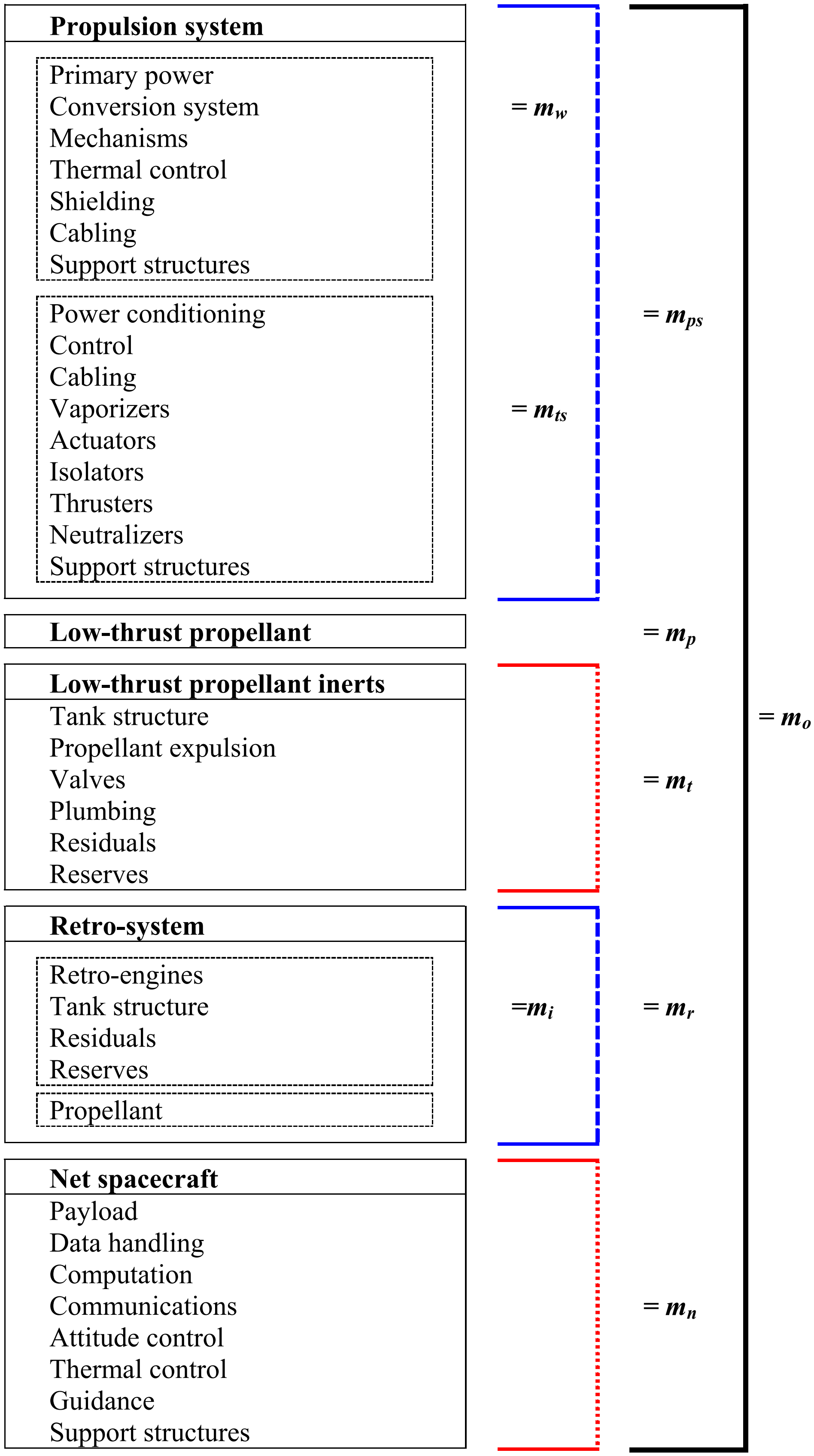
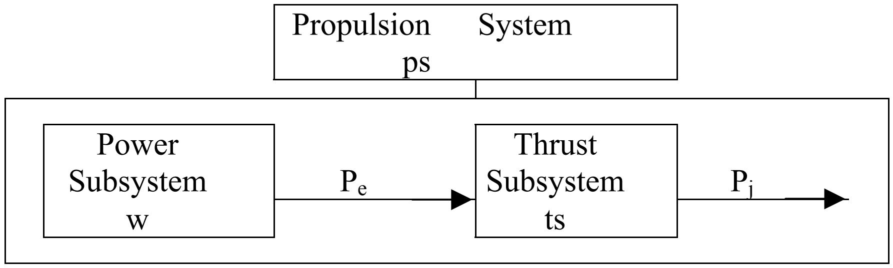


FIGURE 2. System Schematic



retro-propulsion fuel mass which does not include propellant reserves or residuals, and second is the retro-propulsion system inert mass, m_i, which includes the retro-engines, propellant tankage, and fuel reserves. The latter mass is taken to be proportional to the retro-propulsion fuel mass.

From the viewpoint of the mission analyst, the two elements of the low-thrust electric propulsion system of major consequence are thrust magnitude, F, and the propellant mass flow rate $\dot{m}_p$. The thrust acceleration of an electric-propelled spacecraft at any time t is related to its initial mass m_o by the expression

$$a(t) = F(t) / (m_o - \int_o^t \dot{m}_p dt) \qquad \textbf{(1)}$$

EXHAUST JET VELOCITY

We here adopt the convention of defining propulsion exhaust or effective jet velocity as the ratio of thrust to mass flow rate:

$$V_j = F/\dot{m}_p = c \qquad \textbf{(2)}$$

(V_j or c can be used interchangeably for velocity).

Historically, comparative analyses of propulsion systems have been made with a figure of merit defined as “specific impulse”, I_s. This parameter has the dimension of time because of the consideration of propellant "weight" flow rather than mass flow in equation (2). Specific impulse is, therefore, related to jet velocity by the constant of Earth's gravity g_o:

$$I_s = V_j / g_o = c/g_o \qquad \textbf{(3)}$$

Defined in this way, either specific impulse or jet velocity may be used as a figure of merit in the comparison of electric-propulsion thrust systems.

POWER AND EFFICIENCY

The effective jet velocity has been defined as a function of thrust and propellant mass flow rate. It is also possible to define an effective jet power of the propulsion system:

$$P_j = {}^1/_2\, \dot{m}_p V_j^2 \qquad \textbf{(4)}$$

The effective jet power can then be referred to any other point in the propulsion system by introducing an appropriate efficiency function. This efficiency is ordinarily expressed as a function of I_s and is useful in defining electrical power level P_e via the expression

$$\eta_{ts} = P_j / P_e \qquad \textbf{(5)}$$

Further breakdown of this efficiency, which is itself the product of the thruster efficiency and the power conditioning efficiency, into its constituent elements inside the thrust subsystem is of value to hardware developers, but is of little consequence to the mission analyst. There can be interminable dispute regarding the "best" point at which to measure P_e the choice again being of some interest only to hardware developers. We shall here adopt, however arbitrarily, the convention that electrical power P_e is defined as that total electrical power delivered to the input terminals of the power-conditioning assemblies (i.e., into the thrust subsystem) at a fixed reference condition. For nuclear reactor power systems this reference condition is design power level, for radioisotope systems the reference condition is start of mission power, and for solar power systems the reference condition is power at 1 A.U. The power variation of the solar and radioisotope systems should be identified so that the system analyst may design for the maximum and minimum power along the mission.

Electrical power level may sometimes be closely related to the spacecraft initial mass. For this reason, it is useful to define a normalized power level proportional to initial spacecraft mass. Termed "specific power", P^*, this normalization may be expressed as

$$P^* = P_e / m_o \qquad \textbf{(6)}$$

SPECIFIC MASS

A convenient figure of merit of electric propulsion technology is the ratio of propulsion system mass to power. This quantity is defined as specific mass α or α_{ps}:

$$\alpha = m_w + m_{ts} / P = \alpha_{ps} \qquad \textbf{(7)}$$

Specific mass may be expressed in terms of either jet power or electrical power:

$$\alpha_j = m_w + m_{ts} / P_j \qquad \textbf{(8)}$$

$$\alpha_e = m_w + m_{ts} / P_e = \alpha \qquad \textbf{(9)}$$

To avoid confusion, the appropriate subscript could be used; however, from common practice, the lack of a subscript shall only refer to electrical power. The specific mass can also be separated into a power subsystem specific mass α_w and a thrust subsystem specific mass α_{ts}:

$$\alpha_w = m_w / P_e \qquad \textbf{(10)} \qquad\qquad \alpha_{ts} = m_{ts} / P_e \qquad \textbf{(11)}$$

The specific mass of a power system is itself a function of power level. This phenomenon has traditionally been handled by displaying results parametrically for some reasonable range of specific mass. However, alternative approaches using an explicit function to represent the dependence of specific mass upon power may have value in some analyses. For example, the relationship $\alpha = (K_1 + K_2 P_e^N) / P_e$ where K_1, K_2, and N are appropriately chosen constants reflecting technology level, has proved useful in the case of nuclear reactor powered systems.

NOMENCLATURE

a	Thrust acceleration, m/s^2
c *or* V_j	Effective thruster jet or exhaust velocity, m/s
F	Thrust force, N
I_s	Effective thruster specific impulse, seconds
K_n	Net spacecraft structure proportionally constant
k_o	Structural proportionality constant
k_p	Propellant inert proportionality constant
k_r	Retro-system inert proportionality constant
m	Instantaneous mass of spacecraft, kg
m_f	Final spacecraft mass, kg
m_i	Retro-system inert mass, kg
m_L	Payload mass, kg
m_n	Net spacecraft mass, kg
m_o	Initial spacecraft mass, kg
m_p	Low-thrust propellant mass, kg
m_{pr}	Retro-propulsion fuel mass, kg
m_{ps}	Propulsion system[1] mass, kg
m_r	Retro-system mass, kg
m_s	Structural mass, kg
m_t	Low-thrust propellant tankage or inert mass, kg
m_{ts}	Thrust subsystem mass, kg
m_w	Power subsystem mass, kg
P_e	Electrical power to thruster subsystem, kW or kW$_e$ optionally
P_j	Kinetic power in jet exhaust, kW or kW$_j$ optionally
t_f	Mission times (days)
t_p	Propulsion times (hours)
V_j *or* c	Effective thruster jet or exhaust velocity, m/sec
α *or* α_{ps}	Propulsion system specific mass, kg/kW$_e$
α_{ts}	Thruster subsystem specific mass, kg/kW$_e$
α_w	Power subsystem specific mass, kg/kW$_e$
ηt_s	Thrust subsystem efficiency

[1] "Propulsion system" as used here includes neither propellant nor tankage.

TERMINOLOGY

Initial spacecraft mass[1]

$$m_o = m_{ps} + m_p + m_t + m_r + m_n + m_s$$

($m_s = 0$ preferred, see text)

Propulsion system mass

$$m_{ps} = m_w + m_{ts} = \alpha\, P_e$$

Low-thrust propellant tankage or inert mass

$$m_t = k_p\, m_p$$

Retro-system mass including inerts

$$m_r = m_i + m_{pr} = k_p\, m_{pr} + m_{pr}$$

Structural mass[2]

$$m_s = k_o\, m_o \ or\ k_n\, m_n$$

(preferred approach is $m_s = 0$)

Payload mass, final mass

$$m_L \leq m_n$$

(m_L and m_f defined by analyst when used)

Thrust subsystem efficiency

$$\eta_{ts} = P_j / P_e$$

Thruster jet or exhaust velocity

$$V_j = F / \dot{m}_p = c$$

Thruster specific impulse

$$I_s = V_j / g_o = c / g_o$$

(g_o is a defined constant equal to 9.80665 m/s^2)

Thrust acceleration

$$a = F/m\ ;\ a_o = F / m_o$$

Specific mass

$$\alpha_w = m_w / P_e$$

$$\alpha_{ts} = m_{ts} / P_e$$

$$\alpha = m_w + m_{ts} / P_e = \alpha_w + \alpha_{ts} = \alpha_{ps}$$

[1] In some circumstances, a retro-system is not included ($m_r = 0$). Structure may not be accounted for explicitly ($m_r = 0$).

[2] It should be recognized that use of the m_s option can cause a double penalty because of the allocation for structure within $m_{ps}(\alpha)$, $m_t(k_p)$, $m_n(k_n)$, and $m_r(k_r)$.